Pre Calculus
Unit 2 Review Packet

Name: KEY
Date:

This sheet represents the types of questions and concepts you will find on the test. It is not, however, meant to be a replica of the test. Study all notes, homework assignments, quizzes, and this review to prepare adequately for tomorrow. Good luck!

Factor or Solve:

1. $-3x^2 + x + 12 = -4x^2 + 32$

$x^2 + x - 20 = 0$ $\quad x = -5,\ x = 4$

$(x+5)(x-4) = 0$

2. $6x^2 + 19x + 10 = 0$

$6x^2 + 15x + 4x + 10 = 0$ $\quad x = -\frac{2}{3},\ x = -\frac{5}{2}$

$(3x+2)(2x+5) = 0$

3. $12x^2 + 7x + 8 = 2x^2 + 20$

$10x^2 + 7x - 12 = 0$ $\quad x = -\frac{3}{2}$

$(2x+3)(5x-4) = 0$ $\quad x = \frac{4}{5}$

4. $4x^2 + 20x + 25$

$(2x+5)(2x+5)$

$(2x+5)^2$

5. $20x^3 - 5x$

$5x(4x^2 - 1) = 5x(2x+1)(2x-1)$

6. $(x+1)^2 - 4y^2$

$(x+1+2y)(x-1+2y)$

7. $x^3 + 64y^3$

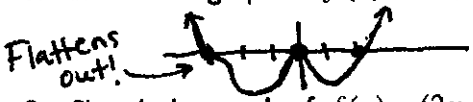
$(x+4y)(x^2 - 4xy + 16y^2)$

8. $343x^3 - 27$

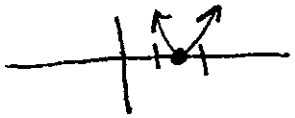
$(7x-3)(49x^2 + 21x + 9)$

Graphs of Polynomials

1. Sketch the graph of $f(x) = x^2(x-2)(x+3)^3$ using end behavior and multiplicity.



2. Sketch the graph of $f(x) = (2x-3)^2(x^2+25)$ using end behavior and multiplicity.



$x = \frac{3}{2} = 1.5$

Deg 4

3. Find the factored form of a 4rd degree polynomial with zeros of 2, -3, and 5.

$f(x) = (x-2)(x+3)(x-5)^2$ (one option of many)

One of the simplest rational functions is the reciprocal function, $f(x) = \frac{1}{x}$.

Describe how the graph of the given functions below can be obtained by transforming the graph of the reciprocal function above. Identify the end-behavior asymptotes, the vertical asymptotes, and then sketch the graph.

1. $g(x) = \frac{2}{x+3}$ · Stretch 2, shift left 3 · H.A. @ $y=0$ · V.A. @ $x=-3$

2. $h(x) = \frac{3x-6}{x-2} = \frac{3(x-2)}{(x-2)}$ · Hole at $x=2$ · H.A. @ 3

Find the important pieces and graph (VA/Holes, HA/~~Slant~~, all intercepts, domain).

1. $f(x) = -\frac{3}{x-1}$ V.A. @ $x=1$; No Holes; H.A. @ $y=0$; $(0,3)$; (No x-int.); D: $(-\infty,1)(1,+\infty)$

2. $g(x) = \frac{5-2x}{x+4}$ V.A.: $x=-4$; No Holes; y-int: $(0, \frac{5}{4})$; H.A. @ $y=-2$; x-int: $(\frac{5}{2}, 0)$; D: $(-\infty,-4)(-4,+\infty)$

3. $h(x) = \frac{2x^2-2}{x^2-4} = \frac{2(x+1)(x-1)}{(x+2)(x-2)}$ V.A. @ $x=2,-2$; H.A. @ $y=2$; int: $(0,\frac{1}{2})$ $(-1,0)$ $(1,0)$; D: $(-\infty,-2)(-2,2)(2,+\infty)$; No Holes

4. $i(x) = \frac{x^3}{x^2-9} = \frac{x^3}{(x+3)(x-3)}$ No H.A. → ends to infinity; V.A. @ $x=-3, x=3$; No Holes; Int: $(0,0)$; D: $(-\infty,-3)(-3,3)(3,+\infty)$

5. $j(x) = \frac{x^2-16}{x+4} = \frac{(x+4)(x-4)}{x+4}$ No V.A.; Hole @ $x=-4$; Int: $(0,-4)$ $(4,0)$; D: $(-\infty,-4)(-4,+\infty)$

Simplify the following rational expressions.

1. $\frac{x^2+4x-21}{x^2-9x+18} = \frac{(x+7)(x-3)}{(x-6)(x-3)}$

2. $\frac{\frac{x^2+7x}{3x}}{\frac{49-x^2}{3x-21}} = \frac{(x+7)\cdot 3\cdot(x-7)}{3(x+7)(x-7)} = 1$

3. $\dfrac{(a+b)\left(\frac{1}{a}-\frac{1}{b}\right)}{(a-b)\left(\frac{1}{a}+\frac{1}{b}\right)} = -1$

4. $\dfrac{\left(\frac{p}{p-4}-\frac{1}{4}\right)}{\left(\frac{9}{4p}+\frac{p^2}{p-4}\right)} = \dfrac{(3p+4)\cdot p}{4p^3+9p-36}$

5. $\dfrac{(a^2-5a+6)^{-1}}{(a-2)^{-2}} \div \dfrac{(a-3)^{-1}}{(a-2)^{-2}} = \dfrac{1}{a-2}$

6. $\dfrac{10}{4y+8}-\dfrac{3}{7y+14} = \dfrac{58}{28(y+2)}$

$= \dfrac{29}{14(y+2)}$

7. $\dfrac{3}{n+4}+\dfrac{4}{n^2-4}$

$\dfrac{3n^2+4n+4}{(n+4)(n^2-4)}$

8. $\dfrac{6}{(a+7)(a-7)}-\dfrac{9}{a-7}$

$\dfrac{6-9(a+7)}{(a+7)(a-7)} = \dfrac{-9a-10}{(a+7)(a-7)}$

Solve each equation. Make sure to verify solutions.

1. $x+\dfrac{3}{x}=4$

$x^2+3=4x$

$x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3$, $x=1$

2. $x+\dfrac{1}{x-4}=0$

can't be factored! Quad Formula!

$x^2-4x+1=0$

$x=2\pm\sqrt{3}$

3. $\dfrac{2x}{x-1}+\dfrac{1}{x-3}=\dfrac{2}{x^2-4x+3}$ $\quad (x-1)(x-3)$

$2x(x-3)+(x-1)=2$

$2x^2-6x+x-1=2$

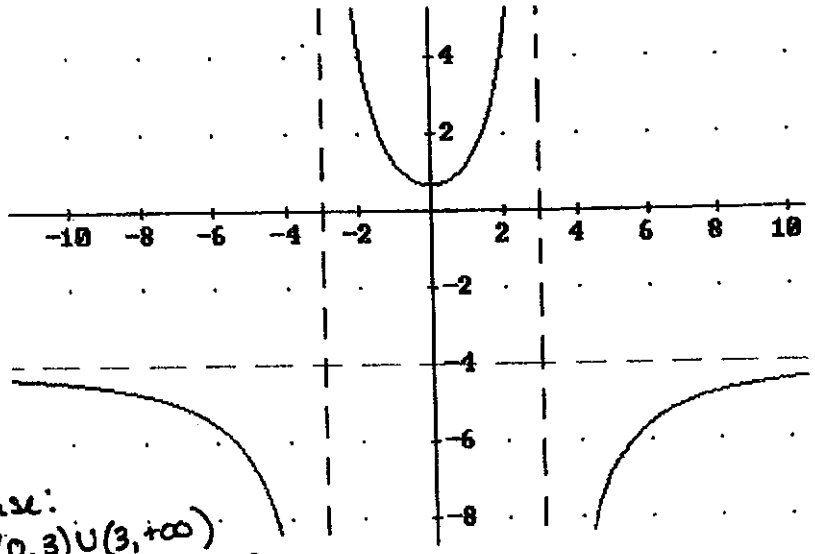
$2x^2-5x-3=0$

$(2x+1)(x-3)=0$

$x=-\frac{1}{2}$, $x=3$ Extraneous!

4. $\dfrac{x-3}{x}+\dfrac{3}{x+2}+\dfrac{6}{x^2+2x}=0$

For the rational function below



increase: $(0,3) \cup (3,+\infty)$

decrease: $(-\infty,3) \cup (3,0)$

1.

a. Give the domain and range. D: $(-\infty,-3) \cup (-3,3) \cup (3,+\infty)$ R: $(-\infty,-4) \cup [1,+\infty)$

c. Give the intervals of increase and decrease.

d. Give any local or absolute extrema (max/mins). $(0,1)$ is a min.

f. Discontinuities (asymptotes, holes)? V.A. @ $x=-3$ & $x=3$

g. Create a possible rational function to fit this graph.

$\frac{-4(x^2+2.25)}{(x-3)(x+3)}$ ← same deg, no x-int, ratio of L.C. = 4, & when $x=0$ $f(0)=\frac{-9}{-9}=1$ for y-int. ← Asymptotes

Create a rational function that satisfies the following characteristics (answers may vary but you can graph your function and see if it satisfies the conditions).

2. x-intercept @ (4,0); vertical asymptote @ x = -3; horizontal asymptote @ y = 2

$\frac{(x-4)}{(x+3)} = f(x)$

3. zero of the function at x = -6; infinite discontinuity at x = 5; ~~end behavior asymptote @ y = x - 2~~ ~~horizontal asymptote~~

$f(x) = \frac{(x+6)}{(x-5)}$

1. Your friend tells you that the functions $f(x) = \frac{x^2-36}{2x+12}$ and $g(x) = \frac{1}{2}(x-6)$ are identical functions. Do you agree with him? If not, how are the functions different and why?

① They behave the same when graphed.

② $f(x)$ simplifies to $\frac{(x-6)}{2} = \frac{1}{2} \cdot (x-6) = g(x)$

③ $f(x) = \frac{(x-6)(x+6)}{2(x+6)}$ has a hole at $x=-6$.

④ They are almost identical except for the hole.

1) -1

2) $\frac{y+2}{y+4}$

3) $y^2 + 5y + 25$

4) $\frac{20x}{7}$

5) $\frac{x}{x-7}$

6) $y - 9$

7) 2

8) $\frac{13x-28}{x(x-7)}$

9) $\frac{-8x-10}{15x}$

10) $\frac{6x^2+10x+21}{(x+7)(x+2)(x-3)}$

11) $\frac{36}{x}$

12) $6x + 5y$

13) ~~$\{x \mid x \neq 5, x \neq 7\}$~~ Domain $(-\infty, 5)(5, 7)(7, +\infty)$

14) all real numbers $(-\infty, +\infty)$

domain: ~~$\{x \mid x \neq 4\}$~~ Domain $(-\infty, 4)(4, +\infty)$

range: ~~$\{y \mid y \neq 5\}$~~ $(-\infty, 5)(5, +\infty)$

16) $x = 7, x = -2$

17) vertical asymptotes: $x = -8, x = 8, x = -7$

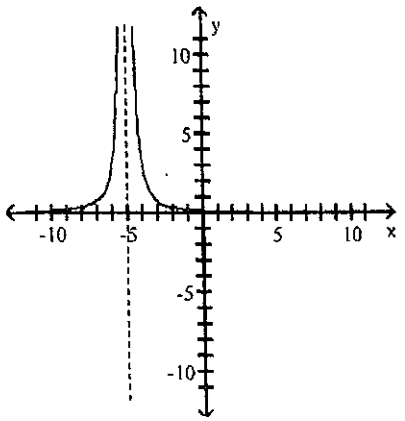
18) $y = \frac{5}{8}$

19) $y = 0$

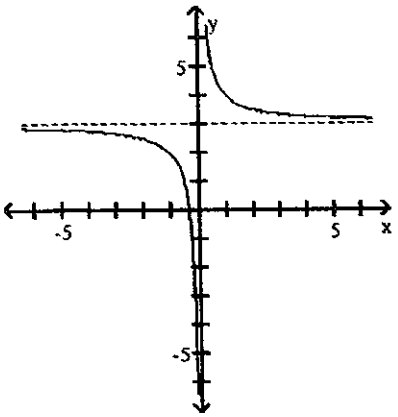
20) $y = x + 6$

21) $y = 2x - 1$

22)



23)



24) (-9, 0)

25) (0, 0) and (-6, 0)

26) $(0, \frac{5}{4})$

27) none

~~28) Yes~~

~~29) $(-\infty, -6)$ or $(6, \infty)$~~

~~30) $(-\infty, -3]$ or $[0, 5)$~~

~~31) $(-6, 7)$~~

~~32) $[-12, 1)$ or $[9, \infty)$~~